Tunnel Interference Assessment from Measurements on Two Interfaces

C. F. Lo*
University of Tennessee Space Institute, Tullahoma, Tennessee

An analytic assessment of two-dimensional wind-tunnel wall interference has been formulated. With measured upwash velocity components along two streamwise interfaces in the tunnel, the Prandtl-Glauert equation for the flowfield is solved by the Fourier transform technique. Analytic formulas for the tunnel-induced interference velocity components at the model are presented. The formulas are verified for analytic models of lifting and blockage interferences induced by the general linear slotted and perforated tunnel wall-boundary conditions. The formulas are also expressed in terms of Fourier coefficients of the measured upwash and then applied to a wavy-wall case to illustrate the merits of the method.

Nomenclature

h = tunnel semiheight
M = source strength
p = Fourier transform parameter
u,v = perturbation velocity components parallel and normal to freestream, respectively (normalized by freestream velocity)
x,y = two-dimensional coordinates parallel and normal to freestream, respectively
β = compressibility
Γ = vortex strength
$(\bar{\ })$ = quantity in transform plane
$|\ |$ = absolute value

Subscripts

A = antisymmetric component
i = interference component
m = model value
s = symmetric component
T = tunnel value
1,2 = interface 1 and 2, respectively
∞ = free-air condition value

I. Introduction

In order to overcome the difficulties of the classical methods in determining wall interference, namely, the mathematical modeling of the ventilated wall configurations and the representation of test articles, a variety of flow variables have been measured near or on the tunnel wall during wind tunnel tests to support wall interference prediction analysis. Furthermore, procedures of recently developed adaptive wall tunnels also require these boundary-flow variable measurements. These boundary measurements are utilized to determine adaptive-wall settings for minimum interference conditions. Techniques using the boundary-flow variable measurements to assess the level of interference supplement the adaptive-wall process in assessing the minimum interference wall settings.

Several wall interference assessment methods using measured flow-boundary conditions are available based on the different kinds of possible measurements, such as two velocity components at a single or double interface.[1-5]

In the present paper, a method is presented for assessing wall interference using one upwash velocity component measured on each of two streamwise interfaces in the tunnel. The method can be applied directly to the new adaptive-wall test section built for NASA Ames 2- by 2-ft Transonic Wind Tunnel.[6] An analytical solution of upwash and pressure interferences at the model will be presented here. This contrasts with a numerical approach available in Ref. 7 for interference assessment using upwash component boundary measurements. The adaptive-wall approach using the measurements of two components along a single interface is preferable under some conditions. The present method is also applicable to the case of two components/one interface, and the work currently is under development. Some other numerical approaches including three-dimensional and transonic effects are available in the literature.[8,9] During adaptive-wall wind tunnel tests, the analytical solution may be superior in the on-line calculation of the wall interference to the numerical approach. Especially, the precalculated influence matrix of the interference formulas will speed up the on-line application to experimental results.

In the present two-dimensional approach, the proposed method assumes that tunnel side-wall interference is not taken into account. In some wind tunnel arrangement, side-wall interference may become a major factor in the tunnel flow.[10,11] However, the present method may be applied to the tunnel having a negligible amount of side-wall effects such as the NASA Ames Research Center 2-ft Adaptive-Wall Tunnel.[6]

The Prandtl-Glauert equation is chosen as the field equation to describe the subsonic flow regime. The linear wall-boundary condition is also assumed in the verification process to preserve the linearity of the boundary-value problem. The Fourier transform technique is utilized to obtain the analytical solution. The advantage of the Fourier transform method will be shown to be that all equations in the transformed plane are reduced to algebraic form and can be manipulated easily. Analytic expressions for interference assessment are given and applied to both lifting and blockage models. The matrix forms with precalculated coefficients for the analytic expressions can be derived to indicate the saving of computation time for on-line applications. The analytic interference expressions have been simplified in terms of Fourier coefficients of the measured variables and applied to a wavy-wall case to demonstrate the advantages of the method.

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*Professor of Aerospace Engineering, Department of Aerospace and Mechanical Engineering.

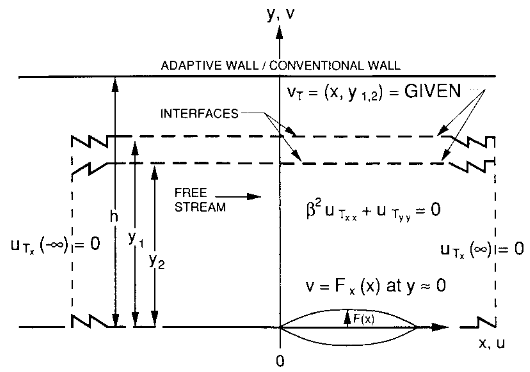

Fig. 1 Boundary value problem of wind tunnel flow with two-interface measurement

II. Formulation of Equation

The boundary-value problem for the tunnel interference as shown in Fig. 1 is formulated by using the Prandtl-Glauert equation and the linearized general slotted and perforated wall-boundary condition. The linearity of the flow characteristics allows that the tunnel flowfield be obtained by the superposition of the interference component to the free-air field. Thus, the upwash component v may be written in the form as

$$v_T(x, \pm y_{1,2}) = v_\infty(x, \pm y_{1,2}) + v_i(x, \pm y_{1,2})$$

$$\bar{v}_T(p, \pm y_{1,2}) = \bar{v}_\infty(p, \pm y_{1,2}) + \bar{v}_i(p, \pm y_{1,2}) \tag{1}$$

where the overbar quantities are represented by the function in the Fourier transformed plane. The boundary-value problem of the wall-induced interference portion can be formulated separately as shown in Fig. 1 because of the linearity of the problem. The interference field is solved by the Fourier transform technique. The relation between the induced upwash $v_i(x,0)$ along the model $y = 0$ and the induced upwash in the field $v_i(x,y)$ is obtained in the Fourier transformed plane as

$$\bar{v}_i(p,0) = \frac{\bar{v}_{i_A}(p,y)}{\cosh p\beta y} \tag{2}$$

where

$$\bar{v}_{i_A}(p,y) = \frac{[\bar{v}_i(p,y) + \bar{v}_i(p,-y)]}{2}$$

and $y \leq h$ is the tunnel semiheight.

It should be noted that there is no requirement to impose either a symmetric or antisymmetric condition on the preceding equation. The antisymmetric quantity v_{i_A} is related to the quantities at $\pm y$ as shown in the Appendix.

Similarly, the upwash components at two interfaces in the free-air field can be expressed in the Fourier transformed plane as

$$\bar{v}_{\infty_A}(p,y_2) = \bar{v}_{\infty_A}(p,y_1)e^{-|p|\beta(|y_2| - |y_1|)} \tag{3}$$

where

$$\bar{v}_{\infty_A}(p,y) = \frac{[\bar{v}_\infty(p,y) + \bar{v}_\infty(p,-y)]}{2}$$

The $\bar{v}_{\infty_A}(p,y_2)$ on the left side of Eq. (3), using Eqs. (1) and (2), may be written as

$$\bar{v}_{\infty_A}(p,y_2) = \bar{v}_{T_A}(p,y_2) - \bar{v}_i(p,0)\cosh p\beta y_2$$

Similarly, $\bar{v}_{\infty_A}(p,y_1)$ on the right side of Eq. (3) may be written as

$$\bar{v}_{\infty_A}(p,y_1) = \bar{v}_{T_A}(p,y_1) - \bar{v}_i(p,0)\cosh p\beta y_1$$

Substituting $\bar{v}_{\infty_A}(p,y_1)$ and $\bar{v}_{\infty_A}(p,y_2)$ in Eq. (3), one obtains the upwash interferences as

$$\bar{v}_i(p,0) = \frac{\bar{v}_{T_A}(p,y_2) - \bar{v}_{T_A}(p,y_1)e^{-|p|\beta(|y_2| - |y_1|)}}{\cosh p\beta y_2 - \cosh p\beta y_1 e^{-|p|\beta(|y_2| - |y_1|)}}$$

where

$$\bar{v}_{T_A}(p,y) = [\bar{v}_T(p,y) + \bar{v}_T(p,-y)]/2$$

The preceding upwash interference equation in terms of tunnel measurements may be further simplified

$$\bar{v}_i(p,0) = \frac{\bar{v}_{T_A}(p,y_2)e^{-|p|\beta y_1} - \bar{v}_{T_A}(p,y_1)e^{-|p|\beta y_2}}{\beta \sinh|p|\beta(y_2 - y_1)} \tag{4}$$

The upwash interference equation $v_i(x,0)$ in the physical plane obtained by the inversion of Eq. (4) using the convolution theorem[12] is expressed in the form of

$$v_i(x,0) = \frac{-2}{\beta}(F_1 + F_2) \tag{5}$$

where

$$F_1 = \frac{1}{\pi^2}\int_{-\infty}^{\infty}\left[\oint_{-\infty}^{\infty}\frac{v_{T_A}(\eta,y_1)}{\xi - \eta}\,d\eta\right]\sum_{k=1}^{\infty}S_{ky_1}(x - \xi)\,d\xi$$

$$F_2 = \frac{-1}{\pi^2}\int_{-\infty}^{\infty}\left[\oint_{-\infty}^{\infty}\frac{v_{T_A}(\eta,y_2)}{\xi - \eta}\,d\eta\right]\sum_{k=1}^{\infty}S_{ky_2}(x - \xi)\,d\xi$$

with

$$v_{T_A}(\xi,y_{1,2}) = [v_T(\xi,y_{1,2}) + v_T(\xi,-y_{1,2})]/2$$

$$S_{ky_1}(x - \xi) = \frac{x - \xi}{[\beta y_1 + (2k-1)\beta(y_2 - y_1)]^2 + (x - \xi)^2}$$

$$S_{ky_2}(x - \xi) = \frac{x - \xi}{[\beta y_2 + (2k-1)\beta(y_2 - y_1)]^2 + (x - \xi)^2}$$

The streamwise velocity interference along the model at $y = 0$ can be derived in a similar way. The expression in the physical plane is of the form

$$u_i(x,0) = G_1 + G_2 \tag{6}$$

where

$$G_1 = \frac{2}{\pi\beta}\int_{-\infty}^{\infty} v_{T_S}(\xi,y_1)\sum_{k=1}^{\infty} S_{ky_2}(x - \xi)\,d\xi$$

$$G_2 = \frac{-2}{\pi\beta}\int_{-\infty}^{\infty} v_{T_S}(\xi,y_2)\sum_{k=1}^{\infty} S_{ky_1}(x - \xi)\,d\xi$$

with

$$v_{T_S}(\xi,y_{1,2}) = [v_T(\xi,y_{1,2}) - v_T(\xi,-y_{1,2})]/2$$

The application of lifting and blockage interferences, Eq. (5) and Eq. (6), may be utilized for the direct numerical series summation and integration. Alternatively, in order to speed up for the on-line calculation, Eq. (6), for example, can be rewritten in a matrix format with the measured velocity $v_{T_A}(y_1)$ as a vector and the integral kernel as a matrix. The elements of the matrix are obtained from the integration quadrature of a selected numerical integration formula, such as Simpson's rule and the integral's kernel, which is a infinite series in Eq. (6). The numerical sum of the infinite series may be obtained by applying Euler Maclaurin's series summation formula, which requires only a limited number of terms. The elements of the matrix are precalculated and stored in the computer's memory. The only on-line computation operation is a matrix multiplication to the measured velocity component vector. In the next section, the application of the formula will be verified for analytic models of lifting and blockage interferences for the general linear slotted and perforated tunnel wall-boundary conditions.

III. Application

The application of lifting and blockage interference of Eqs. (5) and (6) in a general linear wall-boundary condition is illustrated as follows.

Lifting Interference

The upwash component in the wind tunnel with a two-dimensional vortex can be expressed[7] at two interfaces $y_{1,2}$

$$\begin{aligned} v_{T_A}(x,y_{1,2}) &= v_m(x,y_{1,2}) + v_i(x,y_{1,2}) \\ &= \frac{-\Gamma}{2\pi}\beta\frac{x}{x^2+\beta^2y_{1,2}^2} \\ &-\frac{\Gamma}{2\pi h}\left[\int_0^\infty A(R,K,p)\cosh\lambda\beta y_{1,2}\cos\lambda x\,\mathrm{d}p \right. \\ &\left. + \int_0^\infty B(R,K,p)\cosh\lambda\beta y_{1,2}\sin\lambda x\,\mathrm{d}p\right] \end{aligned} \tag{7}$$

where $\lambda = p/\beta h$. The A and B are functions of porosity R and slot parameter K.

The upwash components of Eq. (7) at two interfaces are substituted into Eq. (5). The result reduced to Eq. (8) is the same form as that in Ref. 7, as expected.

$$\begin{aligned} v_i(v,0) &= \frac{-\Gamma}{2\pi h}\left[\int_0^\infty A(R,K,p)\cos\lambda x\,\mathrm{d}p \right. \\ &\left. + \int_0^\infty B(R,K,p)\sin\lambda x\,\mathrm{d}p\right] \end{aligned} \tag{8}$$

Blockage Interference

The upwash component in the wind tunnel with a two-dimensional source can be expressed[7] at two interfaces $y_{1,2}$

$$\begin{aligned} v_{T_S}(x,y_{1,2}) &= \frac{M}{2\pi}\frac{\beta y_{1,2}}{x^2+\beta^2y_{1,2}^2} \\ &+\frac{M}{2\pi}\left[\int_0^\infty C(R,K,p)\sinh\lambda\beta y_{1,2}\cos\lambda x\,\mathrm{d}p \right. \\ &\left. + \int_0^\infty D(R,K,p)\sinh\lambda\beta y_{1,2}\sin\lambda x\,\mathrm{d}p\right] \end{aligned} \tag{9}$$

where $\lambda = p/\beta h$. The C and D are functions of porosity R and slot parameter K.

Similarly, the blockage interference can be calculated by substituting Eq. (9) into Eq. (6), and the same expression is obtained as that in Ref. 6.

$$\begin{aligned} u_i(x,0) &= \frac{M}{2\pi}\left[\int_0^\infty C(R,K,p)\cos\lambda x\,\mathrm{d}p \right. \\ &\left. + \int_0^\infty D(R,K,p)\sin\lambda x\,\mathrm{d}p\right] \end{aligned} \tag{10}$$

IV. Interference Equations in Terms of Fourier Coefficients

The interference equations can be further simplified if the measured upwash on the interfaces may be written in a Fourier series form as

$$v_{T_A}(x,y_{1,2}) = \sum_{m=1}^{\infty}(a_{1,2,m}\sin mx + b_{1,2,m}\cos mx) \tag{11}$$

$$v_{T_S}(x,y_{1,2}) = \sum_{m=1}^{\infty}(c_{1,2,m}\sin mx + d_{1,2,m}\cos mx) \tag{12}$$

The upwash interference expression is obtained in terms of coefficients of Fourier series expansion of the measured upwash components by substituting Eq. (11) and Eq. (12) into Eqs. (5) and Eq. (6), respectively.

$$\begin{aligned} v_i(x,0) &= \frac{1}{2}\sum_{m=1}^{\infty}(a_{1,m}\sin mx + b_{1,m}\cos mx)\frac{e^{-m\beta y_2}}{\sinh m\beta(y_1-y_2)} \\ &+\frac{1}{2}\sum_{m=1}^{\infty}(a_{2,m}\sin mx + b_{2,m}\cos mx)\frac{e^{-m\beta y_1}}{\sinh m\beta(y_1-y_2)} \end{aligned} \tag{13}$$

and

$$\begin{aligned} u_i(x,0) &= \frac{1}{2\beta}\sum_{m=1}^{\infty}(c_{1,m}\cos mx \\ &+ d_{1,m}\sin mx)\frac{e^{-m\beta y_2}}{\sinh m\beta(y_2-y_1)} + \frac{1}{2\beta}\sum_{m=1}^{\infty}(c_{2,m}\cos mx \\ &+ d_{2,m}\sin mx)\frac{e^{-m\beta y-1}}{\sinh m\beta(y_2-y_1)} \end{aligned} \tag{14}$$

Applying the interference expressions Eqs. (13) and (14) to those upwash measurements which can be expressed in the form of Fourier series, one can simply sum up the series involving Fourier coefficients rather than use Eqs. (5) and (6), which require doing both a series summation and an integration. The wavy-wall model in a tunnel is a good illustration utilizing Eqs. (13) and (14) as follows.

V. Wavy-Wall Model Example

Let the wavy-wall model as $F'(x) = \epsilon\lambda\cos\lambda x$. In the free-air flowfield, the upwash component has the form

$$v_\infty(x,y) = \xi\lambda e^{-\lambda\beta y}\cos\lambda x \tag{15}$$

Or in the Fourier series form

$$v_{T_A}(x,y_1) = b_{1,\lambda}\cos\lambda x \tag{16}$$

$$v_{T_A}(x,y_2) = b_{2,\lambda}\cos\lambda x$$

where $b_{1,\lambda} = \epsilon\lambda e^{-\lambda\beta y_1}$ and $b_{2,\lambda} = \epsilon\lambda e^{-\lambda\beta y_2}$. Substituting the coefficients $b_{1,\lambda}$, $b_{2,\lambda}$ into Eq. (13), we obtain no interference as expected in the free-air flow.

In the tunnel flowfield of wavy wall, the upwash component and the streamwise velocity interference are of the forms

$$v_{T_S}(x,y_{1,2}) = \xi\lambda[\cosh\lambda\beta y_{1,2} - G(h)\sinh\lambda\beta y_{1,2}]\cos\lambda x \tag{17}$$

$$u_i(x,0) = \frac{\xi\lambda}{\beta}[G(h)-1]\sin\lambda x \tag{18}$$

We obtain the interference from Eq. (14) by using Eq. (17) and it yields the results of Eq. (18), as expected.

VI. Summary

The analytical expressions obtained for the lifting and blockage interference predictions are presented as functions of the upwash velocity measurements on two streamwise interfaces within a wind tunnel. Two forms of solution are given: one in terms of the distribution of upwash measurements and the other in terms of Fourier series coefficients of upwash measurements. The formulas have been applied successfully to predict the interferences induced on the analytical models of a two-dimensional vortex, a two-dimensional source and a wavy wall by the general linear ventilated tunnel wall-boundary conditions.

Appendix: Reduction of Symmetric and Antisymmetric Problems

Based on thin airfoil theory, the thickness and camber (including incidence) of an airfoil may be treated as symmetric and antisymmetric problems, respectively. The boundary-value problem outlined in Fig. 1 can be divided into symmetrical and antisymmetrical problems. Thus, the velocity components at the control surface can also be divided into symmetrical and antisymmetrical components as

$$u_T(\pm h) = u_{T_S}(\pm h) + u_{T_A}(\pm h) \tag{A1}$$

and

$$v_T(\pm h) = v_{T_S}(\pm h) + v_{T_A}(\pm h) \tag{A2}$$

with the properties of

$$u_{T_S}(h) = u_{T_S}(-h)$$

$$v_{T_S}(h) = -v_{T_S}(-h)$$

$$u_{T_A}(h) = -u_{T_A}(-h)$$

$$v_{T_A}(h) = v_{T_A}(-h)$$

Then the symmetric and antisymmetric perturbation velocities can be written in terms of the velocity components at the control surfaces as

$$u_{T_S}(\pm h) = [u_T(h) + u_T(-h)]/2 \tag{A3}$$

$$v_{T_S}(\pm h) = \pm[v_T(h) - v_T(-h)]/2 \tag{A4}$$

$$u_{T_A}(\pm h) = \pm[u_T(h) - u_T(-h)]/2 \tag{A5}$$

$$v_{T_A}(\pm h) = [v_T(h) + v_T(-h)]/2 \tag{A6}$$

Similarly, the properties of Eqs. (A3–A6) are also applicable to the interference and free-air fields.

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